

Note: u, v, and w are functions of x, a, c, n, are constants. All trigonometric functions use radians. A constant is multiplied to the result of every integration.

General and Basic Integrals

- 1. ∫ f(x) dx = a ∫ f(x) dx
2. ∫ (u v) dx = ∫ u dx ∫ v dx
3. ∫ u dv = uv - ∫ v du
5. ∫ g'(x) dx = ln|g(x)|
6. ∫ [f(x)]^r dx = { [f(x)]^{r+1} / (r+1) if r ≠ -1, ln|f(x)| if r = -1
7. ∫ x^n dx = x^{n+1} / (n+1) (n ≠ -1)
8. ∫ dx/x = ∫ x^{-1} dx = ln|x| (x ≠ 0)
9. ∫ dx/x^n = ∫ x^{-n} dx = x^{1-n} / (1-n) (n ≠ 1)

SIN

- 1. ∫ sin x dx = -cos x
2. ∫ sin ax dx = -1/a cos ax
3. ∫ sin^2 x dx = 1/2 x - 1/4 sin 2x
4. ∫ sin^2 ax dx = 1/2 x - 1/4 sin 2ax
5. ∫ sin^3 x dx = -1/3 sin^2 x cos x - 1/3 sin x cos^2 x + 1/3 x
6. ∫ sin^3 ax dx = -1/n sin^{n-1} x cos x + (n-1)/n ∫ sin^{n-2} x dx
7. ∫ x sin x dx = sin x - x cos x
8. ∫ x sin ax dx = sin ax/a - x cos ax/a
9. ∫ x^2 sin x dx = -x^2 cos x + 2x sin x + 2 cos x
10. ∫ x^n sin x dx = -x^n cos x + n ∫ x^{n-1} cos x dx
11. ∫ sin^{-1} x dx = x sin^{-1} x + √(1-x^2)
12. ∫ sin^{-1} ax dx = x sin^{-1} ax + 1/a √(1-a^2 x^2)
13. ∫ dx/sin^2 ax = -1/a cot ax
14. ∫ sin ax sin bx dx = -sin(a+b)x / (2(a+b)) + sin(a-b)x / (2(a-b))
15. ∫ 1/(1+sin x) dx = 1/a tan(π/4 - ax/2)

COS

- 1. ∫ cos x dx = sin x
2. ∫ cos ax dx = 1/a sin ax
3. ∫ cos^2 x dx = 1/2 x + 1/4 sin 2x
4. ∫ cos^2 ax dx = 1/2 x + 1/4 sin 2ax
5. ∫ cos^3 x dx = 1/4 cos^3 x sin x + 3/8 cos x sin x + 3/8 x
6. ∫ cos^3 ax dx = 1/n cos^{n-1} x sin x + (n-1)/n ∫ cos^{n-2} x dx
7. ∫ x cos x dx = cos x + x sin x
8. ∫ x cos ax dx = cos ax/a + x sin ax/a
9. ∫ x^2 cos x dx = x^2 sin x + 2x cos x - 2 sin x
10. ∫ x^n cos x dx = x^n sin x - n ∫ x^{n-1} sin x dx
11. ∫ cos^{-1} x dx = x cos^{-1} x - √(1-x^2)
12. ∫ cos^{-1} ax dx = x cos^{-1} ax - 1/a √(1-a^2 x^2)
13. ∫ dx/cos^2 ax = 1/a tan ax
14. ∫ cos ax cos bx dx = sin(a+b)x / (2(a+b)) + sin(a-b)x / (2(a-b))
15. ∫ 1/(1+cos x) dx = 1/a tan(ax/2)

TAN

- tan x = sin x / cos x
1. ∫ tan x dx = -ln|cos x|
2. ∫ tan ax dx = -1/a ln|cos ax|
3. ∫ tan^2 x dx = tan x - x
4. ∫ tan^2 ax dx = x - 1/a tan ax
5. ∫ tan^n x dx = 1/(n-1) tan^{n-1} x - ∫ tan^{n-2} x dx
6. ∫ tan^{-1} x dx = x tan^{-1} x - 1/2 ln|x^2+1|
7. ∫ 1/(1+tan x) dx = 1/2 [x + 1/a ln|cos ax + sin ax|]

COT

- cot x = cos x / sin x
1. ∫ cot x dx = ln|sin x|
2. ∫ cot ax dx = 1/a ln|sin ax|
3. ∫ cot^2 x dx = -cot x - x
4. ∫ cot^2 ax dx = -x - 1/a cot ax
5. ∫ cot^n x dx = -1/(n-1) cot^{n-1} x - ∫ cot^{n-2} x dx
6. ∫ cot^{-1} x dx = x cot^{-1} x + 1/2 ln|x^2+1|

SEC

- sec x = 1/cos x
1. ∫ sec x dx = ln|sec x + tan x|
2. ∫ sec ax dx = 1/a ln|sec ax + tan ax|
3. ∫ sec^2 x dx = tan x
4. ∫ sec^3 x dx = 1/2 sec x tan x + 1/2 ln|sec x + tan x|
5. ∫ sec^n x dx = 1/(n-1) sec^{n-2} x tan x + (n-2)/(n-1) ∫ sec^{n-2} x dx
6. ∫ sec^{-1} x dx = x sec^{-1} x - ln|x + √(x^2-1)|

CSC

- csc x = 1/sin x
1. ∫ csc x dx = -ln|csc x + cot x|
2. ∫ csc ax dx = 1/a ln|csc ax - cot ax|
3. ∫ csc^2 x dx = -cot x

- 4. ∫ csc^n x dx = -1/(n-1) csc^{n-2} x cot x + (n-2)/(n-1) ∫ csc^{n-2} x dx
5. ∫ csc^{-1} x dx = x csc^{-1} x + ln|x + √(x^2-1)|

Combined Trig Functions

- 1. ∫ sin x cos x dx = (sin^2 x)/2
2. ∫ sin ax cos bx dx = -cos(a-b)x / (2(a-b)) - cos(a+b)x / (2(a+b))
3. ∫ sec x tan x dx = sec x
4. ∫ csc x cot x dx = -csc x
5a. ∫ sin^m x cos^n x dx = sin^{m-1} x cos^{n-1} x + (n-1)/(m+n) ∫ sin^m x cos^{n-2} x dx
5b. ∫ sin^m x cos^n x dx = -sin^{m-1} x cos^{n+1} x / (m+n) + (m-1)/(m+n) ∫ sin^{m-2} x cos^n x dx
6. ∫ 1/(a sin x + b cos x) dx = 1/(c√(a^2+b^2)) ln|tan(1/2(x+tan^{-1}(b/a)))|

Integrals Involving e^x

- ∫ F(e^x) dx = 1/∫ F(u) du, u = e^x
1. ∫ e^x dx = e^x
2. ∫ e^{ax} dx = 1/a e^{ax}
3. ∫ x e^x dx = x e^x - e^x
4. ∫ x^n e^x dx = e^x (x^n - n x^{n-1} + n(n-1) x^{n-2} - ... + (-1)^{n-1} x + (-1)^n)
5. ∫ x^n e^{ax} dx = x^n e^{ax} / a - n/a ∫ x^{n-1} e^{ax} dx
6. ∫ dx/(a+be^{cx}) = x/a - 1/ac ln|a+be^{cx}|
7. ∫ (a+e^x)/b dx = ax/b + e^x/b ln|b+e^x|
8. ∫ (a+be^{cx}+ce^{2x})/d dx = ax/d + ce^{cx}/d - a-bd+cd^2/d ln|d+e^x|
9. ∫ dx/(e^x-1) = -1/e^x-1
10. ∫ √(1+e^{2x}) dx = 1/2 √(1+e^{2x}) + 1/2 ln|1+√(1+e^{2x})|
11. ∫ 1/√(e^x+a^2) dx = 2/√(e^x+a^2) ln|√(e^x+a^2)-a-x/a|

Integrals Involving a^x

- 1. ∫ a^x dx = a^x / ln a
2. ∫ x a^x dx = x a^x / ln a - a^x / (ln a)^2
3. ∫ x^n a^x dx = x^n a^x / (ln a) - n/(ln a) ∫ x^{n-1} a^x dx
4. ∫ x^n a^x dx = x^n a^x / (ln a) - n/(ln a) ∫ x^{n-1} a^x dx

Integrals Involving ln(x)

- ∫ F(ln x) dx = ∫ F(u) e^u du, u = ln x
1. ∫ ln x dx = -x + x ln x
2. ∫ ln ax dx = -x + x ln ax
3. ∫ (ln x)^2 dx = x(ln x)^2 - 2x ln x + 2x
4. ∫ x^n ln x dx = x^{n+1} ln x / (n+1) - x^{n+1} / (n+1)^2
5. ∫ x^n ln x dx = x^{n+1} ln x / (n+1) - x^{n+1} / (n+1)^2
6. ∫ x^n ln x dx = x^{n+1} ln x / (n+1) - x^{n+1} / (n+1)^2
7. ∫ e^{ax} ln x dx = e^{ax} (a ln x - b cos bx)
8. ∫ e^{ax} cos bx dx = e^{ax} (a cos bx - b sin bx)

Hyperbolic Functions

- Note: sinh x = (e^x - e^{-x})/2, cosh x = (e^x + e^{-x})/2, tanh x = sinh x / cosh x
1. ∫ sinh x dx = cosh x
2. ∫ sinh^2 x dx = 1/4 sinh 2x - 1/4 x
3. ∫ cosh x dx = sinh x
4. ∫ cosh^2 x dx = 1/4 sinh 2x + 1/4 x
5. ∫ tanh x dx = ln|cosh x|
6. ∫ coth x dx = ln|sinh x|
7. ∫ sech x dx = tan^{-1}(sinh x)
8. ∫ sech^2 x dx = tanh x
9. ∫ sech x tanh x dx = -sech x
10. ∫ sech x coth x dx = -csech x
11. ∫ sech x dx = ln|tanh(1/2)x|
12. ∫ e^{ax} sinh bx dx = e^{ax} (a sinh bx - b cosh bx)
13. ∫ e^{ax} cosh bx dx = e^{ax} (a cosh bx - b sinh bx)

Integrals Involving a+bx

- ∫ F(a+bx) dx = 1/b ∫ F(u) du, u = a+bx
1. ∫ 1/(a+bx) dx = 1/b ln|a+bx|
2. ∫ 1/(a+bx)^2 dx = -1/(a+bx)
3. ∫ 1/(a+bx)^n dx = -1/((n-1)b(a+bx)^{n-1}) (n ≠ 1)
4. ∫ x/(a+bx) dx = 1/b [a+bx - a ln|a+bx|]
5. ∫ x/(a+bx)^2 dx = 1/b [a+bx + ln|a+bx|]
6. ∫ x/(a+bx)^3 dx = -1/(2b^2(a+bx)^2)
7. ∫ x^2/(a+bx) dx = 1/b [1/2(a+bx)^2 - 2a(a+bx) + a^2 ln|a+bx|]
8. ∫ x^2/(a+bx)^2 dx = 1/b [a+bx - a^2/(a+bx) - 2a ln|a+bx|]
9. ∫ (a+bx)^n dx = (a+bx)^{n+1} / (b(n+1)) (n ≠ -1)
10. ∫ 1/(a+bx) dx = 1/b ln|a+bx|
11. ∫ 1/(a+bx) dx = -1/a + b/a ln|a+bx/x|
12. ∫ 1/(a+bx)^2 dx = -1/(a+bx) + 1/a ln|a+bx/x|

Integrals Involving √(a+bx)

- ∫ F(√(a+bx)) dx = 2/√(a+bx) ∫ F(u) du, u = √(a+bx)
∫ F(√(a+bx)) dx = 2/√(a+bx) ∫ F(u) du, u = √(a+bx)
∫ F(√(a+bx)) dx = 2/√(a+bx) ∫ F(u) du, u = √(a+bx)

- 1. ∫ √(a+bx) dx = 2/(3b) (a+bx)^{3/2}
2. ∫ x√(a+bx) dx = 2/(15b^2) (3bx-2a)(a+bx)^{3/2}
3. ∫ x^n √(a+bx) dx = 2x^{n+1} (a+bx)^{3/2} / (b(2n+3)) - 2an / (b(2n+3)) x^{n-1} √(a+bx)
4. ∫ x/(a+bx) dx = 2/(3b^2) (bx-2a)√(a+bx)
5. ∫ x^n/(a+bx) dx = 2x^{n+1} √(a+bx) / (b(2n+1)) - 2an / (b(2n+1)) ∫ x^{n-1}/√(a+bx) dx
6. ∫ 1/(x√(a+bx)) dx = { 1/√a ln|√(a+bx)-√a| for (a>0), 2/√(-a) tan^{-1}√(a+bx)/√(-a) for (a<0)
7. ∫ 1/(x^2√(a+bx)) dx = -√(a+bx)/(a(n-1)x^{n-1}) - b(2n-3)/(2a(n-1)) ∫ 1/(x^{n-1}√(a+bx)) dx
8. ∫ √(a+bx)/x dx = 2√(a+bx) + a ∫ 1/(x√(a+bx)) dx
9. ∫ √(a+bx)/x^2 dx = -(a+bx)^{3/2} / (a(n-1)x^{n-1}) - b(2n-5)/(2a(n-1)) ∫ √(a+bx)/x^{n-1} dx
10. ∫ 1/(√(a+bx)√(c+dx)) dx = 2/√ac tanh^{-1}(c/(ax+b))

Integrals Involving a^2-x^2

- ∫ F(√(a^2-x^2)) dx = a ∫ F(a cos u) cos u du, x = a sin u
1. ∫ 1/(a^2-x^2) dx = 1/a ln|(a+x)/(a-x)|
2. ∫ √(a^2-x^2) dx = 1/2 √(a^2-x^2) + 1/2 a^2 sin^{-1}(x/a)
3. ∫ x√(a^2-x^2) dx = -1/3 (a^2-x^2)^{3/2}
4. ∫ x^2√(a^2-x^2) dx = 1/8 (2x^2-a^2)√(a^2-x^2) + a^4/8 sin^{-1}(x/a)
5. ∫ √(a^2-x^2)/x dx = √(a^2-x^2) - a ln|a+√(a^2-x^2)|
6. ∫ √(a^2-x^2)/x^2 dx = -1/x √(a^2-x^2) - sin^{-1}(x/a)
7. ∫ 1/(a^2-x^2) dx = sin^{-1}(x/a)
8. ∫ x/(a^2-x^2) dx = -√(a^2-x^2)
9. ∫ √(a^2-x^2)/x^2 dx = x/2 √(a^2-x^2) + a^2/2 sin^{-1}(x/a)
10. ∫ 1/(x√(a^2-x^2)) dx = -1/a ln|(a+√(a^2-x^2))/x|
11. ∫ 1/(x^2√(a^2-x^2)) dx = 1/a^2 √(a^2-x^2)
12. ∫ (a^2-x^2)^{3/2} dx = -1/8 (2x^2-5a^2)√(a^2-x^2) + 3a^4/8 sin^{-1}(x/a)
13. ∫ 1/(a^2-x^2)^{3/2} dx = x/(a^2√(a^2-x^2))

Integrals Involving x^2±a^2

- Note: ln|x+√(x^2+a^2)| = sinh^{-1}(x/a)
ln|x+√(x^2-a^2)| = cosh^{-1}(x/a)
∫ F(√(x^2+a^2)) dx = a ∫ F(a sec u) sec^2 u du, x = a tan u
∫ F(√(x^2-a^2)) dx = a ∫ F(a tan u) sec u tan u du, x = a sec u
1. ∫ 1/(x^2+a^2) dx = 1/a tan^{-1}(x/a)
2. ∫ 1/(x^2-a^2) dx = 1/a ln|(x-a)/(x+a)|
3. ∫ x/(x^2+a^2) dx = 1/2 ln|x^2+a^2|
4. ∫ √(x^2+a^2) dx = 1/2 √(x^2+a^2) + 1/2 a^2 ln|x+√(x^2+a^2)|
5. ∫ x√(x^2+a^2) dx = 1/3 (x^2+a^2)^{3/2}
6. ∫ x^2√(x^2+a^2) dx = 1/8 (2x^2+a^2)√(x^2+a^2) - a^4/8 ln|x+√(x^2+a^2)|
7. ∫ √(x^2+a^2)/x dx = √(x^2+a^2) - a sinh^{-1}(x/a)
8. ∫ √(x^2-a^2)/x dx = √(x^2-a^2) - a sec^{-1}|x/a|
9. ∫ √(x^2+a^2)/x^2 dx = -√(x^2+a^2)/x + ln|x+√(x^2+a^2)|
10. ∫ 1/(√(x^2+a^2)) dx = ln|x+√(x^2+a^2)|
11. ∫ 1/(x^2√(x^2+a^2)) dx = -√(x^2+a^2)/a^2
12. ∫ x/(x^2√(x^2+a^2)) dx = 1/a sinh^{-1}(x/a)
13. ∫ 1/(x√(x^2-a^2)) dx = 1/a sec^{-1}|x/a|
14. ∫ x/(√(x^2+a^2)) dx = √(x^2+a^2)
15. ∫ x^2/(√(x^2+a^2)) dx = 1/2 √(x^2+a^2) + 1/2 a^2 ln|x+√(x^2+a^2)|
16. ∫ (x^2+a^2)^{3/2} dx = 1/8 (2x^2+5a^2)√(x^2+a^2) + 3a^4/8 ln|x+√(x^2+a^2)|
17. ∫ 1/(x^2±a^2)^{3/2} dx = ±x/(a^2√(x^2±a^2))

Integrals Involving ax^2+bx+c

- 1. ∫ 1/(ax^2+c) dx = 1/√ac tan^{-1}(x/√(c/a))
2a. ∫ 1/(ax^2+bx+c) dx = 2/√(4ac-b^2) tan^{-1}(2ax+b/√(4ac-b^2)) for 4ac-b^2>0
2b. ∫ 1/(ax^2+bx+c) dx = 1/√(b^2-4ac) ln|(2ax+b-√(b^2-4ac))/(2ax+b+√(b^2-4ac))| for b^2-4ac>0

- 2c. ∫ 1/(ax^2+bx+c) dx = 2/(2ax+b) - b^2-4ac = 0

Integrals Involving a^2±b^2x^2

- 1. ∫ 1/(a^2+b^2x^2) dx = 1/a tan^{-1}(bx/a) (a>0, b>0)
2. ∫ 1/(a^2-b^2x^2) dx = 1/a tan^{-1}(bx/a) = 1/(2ab) ln|(a+bx)/(a-bx)| for (a>0, b>0)
3. ∫ √(a^2+b^2x^2) dx = 1/2 √(a^2+b^2x^2) + a^2/2b ln|(bx/a+√(1+b^2x^2/a^2))|
4. ∫ 1/√(a^2+b^2x^2) dx = 1/b ln|(bx/a+√(1+b^2x^2/a^2))|
5. ∫ √(a^2-b^2x^2) dx = 1/2 √(a^2-b^2x^2) + a^2/2b sin^{-1}(bx/a)
6. ∫ 1/√(a^2-b^2x^2) dx = 1/b sin^{-1}(bx/a)

Integrals Involving √(2ax-x^2)

- 1. ∫ √(2ax-x^2) dx = x/2 √(2ax-x^2) + a^2/2 cos^{-1}(1-x/a)
2. ∫ x√(2ax-x^2) dx = 2x^2-ax-3a^2/6 √(2ax-x^2) + a^3/2 cos^{-1}(1-x/a)
3. ∫ √(2ax-x^2)/x dx = √(2ax-x^2) + a cos^{-1}(1-x/a)
4. ∫ √(2ax-x^2)/x^2 dx = -2√(2ax-x^2)/x - cos^{-1}(1-x/a)
5. ∫ 1/√(2ax-x^2) dx = cos^{-1}(1-x/a)
6. ∫ x/√(2ax-x^2) dx = -√(2ax-x^2) + a cos^{-1}(1-x/a)
7. ∫ x^2/√(2ax-x^2) dx = -x+3a/2 √(2ax-x^2) + 3a^2/2 cos^{-1}(1-x/a)
8. ∫ 1/x√(2ax-x^2) dx = -√(2ax-x^2)/ax
9. ∫ 1/(2ax-x^2)^{3/2} dx = x-a/√(2ax-x^2)

Miscellaneous Integrals

- 1. ∫ x/(a-x) dx = -2a+x/2 √(2ax-x^2) + a^2 sin^{-1}x/a
2. ∫ √(x-a) dx = 2/3 ln|x^3+√(x^3-a)|
3. ∫ √(x+a) dx = √(x+a) - a ln|√(x+a)+√x|
4. ∫ 1/(1-x^2)√(1+x^2) dx = 1/√2 ln|√(2+x^2)+√(1+x^2)|

DERIVATIVES

- dw/dx = dw/du * du/dx : Chain Rule
du/dx = (du/dt) * dt/dx
d/dx(f(u)) = df/du * du/dx
d/dx(uv) = u dv/dx + v du/dx
d/dx(u/v) = (v du/dx - u dv/dx)/v^2
d/dx(u^w) = w u^{w-1} du/dx + u^w ln u du/dx
d/dx ln x = 1/x
d/dx a^x = a^x ln a
d/dx a^{u(x)} = a^u ln a du/dx
d(log_e u)/dx = (log_e e) du/dx
d(sin x)/dx = cos x
d(tan x)/dx = sec^2 x
d(sec x)/dx = sec x tan x
d(csc x)/dx = -csc x cot x
d(cos^{-1} u)/dx = -1/√(1-u^2) du/dx for (-π/2 ≤ sin^{-1} u ≤ π/2)
d(cos^{-1} u)/dx = -1/√(1-u^2) du/dx for (0 ≤ cos^{-1} u ≤ π)
d(tan^{-1} u)/dx = 1/(1+u^2) du/dx for (-π/2 < tan^{-1} u < π/2)
d(cot^{-1} u)/dx = -1/(1+u^2) du/dx for (0 < cot^{-1} u < π)
d(sec^{-1} u)/dx = 1/(u√(u^2-1)) du/dx for (0 ≤ sec^{-1} u < π/2) ∪ (π ≤ sec^{-1} u < 3π/2)
d(csc^{-1} u)/dx = -1/(u√(u^2-1)) du/dx for (0 < csc^{-1} u ≤ π/2) ∪ (-π < csc^{-1} u ≤ -π/2)
d(sinh x)/dx = cosh x
d(tanh x)/dx = sech^2 x
d(sech x)/dx = -sech x tanh x
d(csch x)/dx = -csch x coth x
d(sinh^{-1} u)/dx = 1/√(u^2+1) du/dx
d(cosh^{-1} u)/dx = 1/√(u^2-1) du/dx for u>1 and cosh^{-1} u ≥ 0
d(tanh^{-1} u)/dx = 1/(1-u^2) du/dx for -1 < u < 1