

Note: u, v, and w are functions of x. a, c, n, are constants. All trigonometric functions use radians. A constant must be added to the result of every integration.

General and Basic Integrals

1. $\int af(x)dx = a\int f(x)dx$

2. $\int (u \pm v)dx = \int udx \pm \int vdx$

3. $\int udv = uv - \int vdu$

5. $\int \frac{g'(x)}{g(x)}dx = \ln|g(x)|$

6. $\int [g(x)]^r g'(x)dx = \begin{cases} \{[g(x)]^{r+1}\}/(r+1) & r \neq 1 \\ \ln|g(x)| & r = 1 \end{cases}$

7. $\int x^n dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$

8. $\int \frac{dx}{x} = \int x^{-1}dx = \ln|x| \quad (x \neq 0)$

9. $\int \frac{dx}{x^n} = \int x^{-n}dx = \frac{x^{1-n}}{1-n} \quad (n \neq 1)$

SIN

1. $\int \sin x dx = -\cos x$

2. $\int \sin ax dx = -\frac{1}{a}\cos ax$

3. $\int \sin^2 x dx = \frac{1}{2}x - \frac{1}{4}\sin 2x$

4. $\int \sin^2 ax dx = \frac{1}{2}x - \frac{1}{4}\frac{\sin 2ax}{a}$

5. $\int \sin^4 x dx = -\frac{1}{4}\sin^3 x\cos x - \frac{3}{8}\sin x\cos x + \frac{3}{8}x$

6. $\int \sin^n x dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x dx$

7. $\int x\sin x dx = \sin x - x\cos x$

8. $\int x\sin ax dx = \frac{\sin ax}{a^2} - \frac{x\cos ax}{a}$

9. $\int x^2\sin x dx = -x^2\cos x + 2x\sin x + 2\cos x$

10. $\int x^n\sin x dx = -x^n\cos x + n\int x^{n-1}\cos x dx$

11. $\int \sin^{-1}x dx = x\sin^{-1}x + \sqrt{1-x^2}$

12. $\int \sin^{-1}ax dx = x(\sin^{-1}ax) + \frac{1}{a}\sqrt{1-a^2x^2}$

13. $\int \frac{dx}{\sin^2 ax} = -\frac{1}{a}\cot ax$

14. $\int \sin ax\sin bx dx = -\frac{\sin(a+b)x}{2(a+b)} + \frac{\sin(a-b)x}{2(a-b)}$

15. $\int \frac{1}{1+\sin x}dx = \frac{1}{a}\tan\left(\frac{\pi}{4} - \frac{ax}{2}\right)$

COS

1. $\int \cos x dx = \sin x$

2. $\int \cos ax dx = \frac{1}{a}\sin ax$

3. $\int \cos^2 x dx = \frac{1}{2}x + \frac{1}{4}\sin 2x$

4. $\int \cos^2 ax dx = \frac{1}{2}x + \frac{1}{4}\frac{\sin 2ax}{a}$

5. $\int \cos^4 x dx = \frac{1}{4}\cos^3 x\sin x + \frac{3}{8}\cos x\sin x + \frac{3}{8}x$

6. $\int \cos^n x dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x dx$

7. $\int x\cos x dx = \cos x + x\sin x$

8. $\int x\cos ax dx = \frac{\cos ax}{a^2} + \frac{x\sin ax}{a}$

9. $\int x^2\cos x dx = x^2\sin x + 2x\cos x - 2\sin x$

10. $\int x^n\cos x dx = x^n\sin x - n\int x^{n-1}\sin x dx$

11. $\int \cos^{-1}x dx = x\cos^{-1}x - \sqrt{1-x^2}$

12. $\int \cos^{-1}ax dx = x(\cos^{-1}ax) - \frac{1}{a}\sqrt{1-a^2x^2}$

13. $\int \frac{dx}{\cos^2 ax} = \frac{1}{a}\tan ax$

14. $\int \cos ax\cos bx dx = \frac{\sin(a+b)x}{2(a+b)} + \frac{\sin(a-b)x}{2(a-b)}$

15. $\int \frac{1}{1+\cos ax}dx = \frac{1}{a}\tan\frac{ax}{2}$

TAN $\tan x = \frac{\sin x}{\cos x}$

1. $\int \tan x dx = -\ln|\cos x|$

2. $\int \tan ax dx = -\frac{1}{a}\ln(\cos ax)$

3. $\int \tan^2 x dx = \tan x - x$

4. $\int \tan^2 ax dx = -x + \frac{1}{a}\tan ax$

5. $\int \tan^n x dx = \frac{1}{n-1}\tan^{n-1}x - \int \tan^{n-2}x dx$

6. $\int \tan^{-1}x dx = x\tan^{-1}x - \frac{1}{2}\ln(x^2+1)$

7. $\int \frac{1}{1+\tan ax}dx = \frac{1}{2}\left[x + \frac{1}{a}\ln(\cos ax + \sin ax)\right]$

COT $\cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$

1. $\int \cot x dx = \ln|\sin x|$

2. $\int \cot ax dx = \frac{1}{a}\ln(\sin ax)$

3. $\int \cot^2 x dx = -\cot x - x$

4. $\int \cot^2 ax dx = -x - \frac{1}{a}\cot ax$

5. $\int \cot^n x dx = -\frac{1}{n-1}\cot^{n-1}x - \int \cot^{n-2}x dx$

6. $\int \cot^{-1}x dx = x\cot^{-1}x + \frac{1}{2}\ln(x^2+1)$

SEC $\sec x = \frac{1}{\cos x}$

1. $\int \sec x dx = \ln|\sec x + \tan x|$

2. $\int \sec ax dx = \frac{1}{a}\ln(\sec ax + \tan ax)$

3. $\int \sec^2 x dx = \tan x$

4. $\int \sec^3 x dx = \frac{1}{2}\sec x\tan x + \frac{1}{2}\ln|\sec x + \tan x|$

5. $\int \sec^n x dx = \frac{1}{n-1}\sec^{n-2}x\tan x + \frac{n-2}{n-1}\int \sec^{n-2}x dx$

6. $\int \sec^{-1}x dx = x\sec^{-1}x - \ln|x + \sqrt{x^2-1}|$

CSC $\csc x = \frac{1}{\sin x}$

1. $\int \csc x dx = -\ln|\csc x + \cot x|$

2. $\int \csc ax dx = \frac{1}{a}\ln(\csc ax - \cot ax)$

3. $\int \csc^2 x dx = -\cot x$

4. $\int \csc^n x dx = -\frac{1}{n-1}\csc^{n-2}x\cot x + \frac{n-2}{n-1}\int \csc^{n-2}x dx$

5. $\int \csc^{-1}x dx = x\csc^{-1}x + \ln|x + \sqrt{x^2-1}|$

Combined Trig Functions

1. $\int \sin x\cos x dx = (\sin^2 x)/2$

2. $\int \sin ax\cos bx dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)}$

3. $\int \sec x\tan x dx = \sec x$

4. $\int \csc x\cot x dx = -\csc x$

5a. $\int \sin^m x\cos^n x dx = \frac{\sin^{m+1}x\cos^{n-1}x}{m+n} + \frac{n-1}{m+n}\int \sin^m x\cos^{n-2}x dx$

$\int \sin^m x\cos^n x dx = -\frac{\sin^{m-1}x\cos^{n+1}x}{m+n} + \frac{m-1}{m+n}\int \sin^{m-2}x\cos^n x dx$

6. $\int \frac{1}{a\sin cx + b\cos cx}dx = \frac{1}{c\sqrt{a^2+b^2}}\ln\left[\tan\frac{1}{2}\left(cx + \tan^{-1}\frac{b}{a}\right)\right]$

Integrals Involving e^x e^{ax}

$\int F(e^{ax})dx = \frac{1}{a}\int \frac{F(u)}{u}du, u = e^{ax}$

1. $\int e^x dx = e^x$

2. $\int e^{ax}dx = \frac{1}{a}e^{ax}$

3. $\int xe^x dx = xe^x - e^x$

4. $\int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$

5. $\int x^n e^x dx = x^n e^x - n\int x^{n-1}e^x dx$

6. $\int \frac{dx}{a+be^{cx}} = \frac{x}{a} - \frac{1}{ac}\ln(a+be^{cx})$

7. $\int \frac{a+e^x}{b+e^x}dx = \frac{ax}{b} - \frac{a-b}{b}\ln(b+e^x)$

8. $\int \frac{a+be^x+ce^{2x}}{d+e^x}dx = \frac{ax}{d} + ce^x - \frac{a-bd+cd^2}{d}\ln(d+e^x)$

9. $\int \frac{x}{(e^x-1)^2}dx = -\frac{1}{2}\frac{e^x+1}{e^x-1}$

10. $\int \sqrt{1+e^{ax}}dx = \frac{2}{a}\sqrt{1+e^{ax}} + \frac{1}{a}\ln\frac{\sqrt{1+e^{ax}}+1}{\sqrt{1+e^{ax}}-1}$

11. $\int \frac{1}{\sqrt{e^x+a^2}}dx = \frac{2}{a}\ln(\sqrt{e^x+a^2}-a) - \frac{x}{a}$

Integrals Involving a^x x^n

1. $\int a^x dx = \frac{a^x}{\ln a}$

2. $\int x^n a^x dx = \frac{x^n a^x}{\ln a} - \frac{n}{\ln a}\int x^{n-1}a^x dx$

3. $\int \frac{x^n}{\sqrt{x^3+1}}dx = \frac{2}{2n-1}x^{n-2}\sqrt{x^3+1} - \frac{2n-4}{2n-1}\int \frac{x^{n-3}}{\sqrt{x^3+1}}dx$

4. $\int \frac{x^n}{\sqrt{x^4+1}}dx = \frac{x^{n-3}}{n-1}\sqrt{x^4+1} - \frac{n-3}{n-1}\int \frac{x^{n-4}}{\sqrt{x^4+1}}dx$

Integrals Involving $\ln(x)$

$\int F(\ln x)dx = \int F(u)e^u du, u = \ln x$

1. $\int \ln x dx = -x + x\ln x$

2. $\int \ln ax dx = -x + x\ln ax$

3. $\int (\ln x)^n dx = x(\ln x)^n - n\int (\ln x)^{n-1}dx$

4. $\int \frac{1}{x}dx = \ln|x|$

5. $\int \frac{1}{x\ln x}dx = \ln|\ln x|$

6. $\int x^n \ln x dx = \frac{1}{n+1}x^{n+1}\ln x - \frac{1}{(n+1)^2}x^{n+1}$

7. $\int e^{ax}\sin bx dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$

8. $\int e^{ax}\cos bx dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx - b\sin bx)$

Hyperbolic Functions

Note: $\sinh x = \frac{e^x - e^{-x}}{2}$, $\cosh x = \frac{e^x + e^{-x}}{2}$, $\tanh x = \frac{\sinh x}{\cosh x}$

1. $\int \sinh x dx = \cosh x$

2. $\int \sinh^2 x dx = \frac{1}{4}\sinh 2x - \frac{1}{2}x$

3. $\int \cosh x dx = \sinh x$

4. $\int \cosh^2 x dx = \frac{1}{4}\sinh 2x + \frac{1}{2}x$

5. $\int \tanh x dx = \ln(\cosh x)$

6. $\int \coth x dx = \ln|\sinh x|$

7. $\int \operatorname{sech} x dx = \tan^{-1}(\sinh x)$

8. $\int \operatorname{sech}^2 x dx = \tanh x$

9. $\int \operatorname{sech} x\tanh x dx = -\operatorname{sech} x$

10. $\int \operatorname{csch} x\coth x dx = -\operatorname{csch} x$

11. $\int \operatorname{csch} x dx = \ln\left|\tanh\frac{1}{2}x\right|$

12. $\int e^{ax}\sinh bx dx = \frac{e^{ax}}{a^2-b^2}(a\sinh bx - b\cosh bx)$

13. $\int e^{ax}\cosh bx dx = \frac{e^{ax}}{a^2-b^2}(a\cosh bx - b\sinh bx)$

Integrals Involving $a + bx$

$\int F(a+bx)dx = \frac{1}{b}\int F(u)du, u = a + bx$

1. $\int \frac{1}{a+bx}dx = \frac{1}{b}\ln|a+bx|$

2. $\int \frac{1}{(a+bx)^2}dx = \frac{-1}{b(a+bx)}$

3. $\int \frac{1}{(a+bx)^n}dx = \frac{-1}{(n-1)b(a+bx)^{n-1}} \quad (n \neq 1)$

4. $\int \frac{x}{a+bx}dx = \frac{1}{b^2}[a+bx - a\ln|a+bx|]$

5. $\int \frac{x}{(a+bx)^2}dx = \frac{1}{b^2}\left[\frac{a}{a+bx} + \ln|a+bx|\right]$

6. $\int \frac{x}{(a+bx)^3}dx = -\frac{a+2bx}{2b^2(a+bx)^2}$

7. $\int \frac{x^2}{a+bx}dx = \frac{1}{b^3}\left[\frac{1}{2}(a+bx)^2 - 2a(a+bx) + a^2\ln|a+bx|\right]$

8. $\int \frac{x^2}{(a+bx)^2}dx = \frac{1}{b^3}\left[a+bx - \frac{a^2}{a+bx} - 2a\ln|a+bx|\right]$

9. $\int (a+bx)^n dx = \frac{(a+bx)^{n+1}}{b(n+1)} \quad (n \neq -1)$

10. $\int \frac{1}{x(a+bx)}dx = \frac{1}{a}\ln\left|\frac{x}{a+bx}\right|$

11. $\int \frac{1}{x^2(a+bx)}dx = -\frac{1}{ax} + \frac{b}{a^2}\ln\left|\frac{a+bx}{x}\right|$

12. $\int \frac{1}{x(a+bx)^2}dx = \frac{1}{a(a+bx)} + \frac{1}{a^2}\ln\left|\frac{x}{a+bx}\right|$

Integrals Involving $\sqrt{a+bx}$

$\int F(\sqrt{a+bx})dx = \frac{2}{b}\int uF(u)du, u = \sqrt{a+bx}$

$\int F(\sqrt[n]{a+bx})dx = \frac{n}{b}\int u^{n-1}F(u)du, u = \sqrt[n]{a+bx}$

1. $\int \sqrt{a+bx}dx = \frac{2}{3b}(a+bx)^{3/2}$

2. $\int x\sqrt{a+bx}dx = \frac{2}{15b^2}(3bx-2a)(a+bx)^{3/2}$

3. $\int x^n\sqrt{a+bx}dx = \frac{2x^n(a+bx)^{3/2}}{b(2n+3)} - \frac{2an}{b(2n+3)}\int x^{n-1}\sqrt{a+bx}dx$

4. $\int \frac{x}{\sqrt{a+bx}}dx = \frac{2}{3b^2}(bx-2a)\sqrt{a+bx}$

5. $\int \frac{x^n}{\sqrt{a+bx}}dx = \frac{2x^n\sqrt{a+bx}}{b(2n+1)} - \frac{2an}{b(2n+1)}\int \frac{x^{n-1}}{\sqrt{a+bx}}dx$

6. $\int \frac{1}{x\sqrt{a+bx}}dx = \begin{cases} \frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+bx}+\sqrt{a}}\right| & \text{for } (a>0) \\ \frac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\frac{a+bx}{-a}} & \text{for } (a<0) \end{cases}$

7. $\int \frac{1}{x^n\sqrt{a+bx}}dx = -\frac{\sqrt{a+bx}}{a(n-1)x^{n-1}} - \frac{b(2n-3)}{2a(n-1)}\int \frac{1}{x^{n-1}\sqrt{a+bx}}dx$

8. $\int \frac{\sqrt{a+bx}}{x}dx = 2\sqrt{a+bx} + a\int \frac{1}{x\sqrt{a+bx}}dx$

9. $\int \frac{\sqrt{a+bx}}{x^n}dx = -\frac{(a+bx)^{3/2}}{a(n-1)x^{n-1}} - \frac{b(2n-5)}{2a(n-1)}\int \frac{\sqrt{a+bx}}{x^{n-1}}dx$

10. $\int \frac{1}{\sqrt{ax+b}\sqrt{cx+d}}dx = \frac{2}{\sqrt{ac}}\tanh^{-1}\sqrt{\frac{c(ax+b)}{a(cx+d)}}$

Integrals Involving $a^2 - x^2$ $\sqrt{a^2 - x^2}$

$\int F(\sqrt{a^2-x^2})dx = a\int F(a\cos u)\cos u du, x = a\sin u$

1. $\int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right|$

2. $\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$

3. $\int x\sqrt{a^2-x^2}dx = -\frac{1}{3}(a^2-x^2)^{3/2}$

4. $\int x^2\sqrt{a^2-x^2}dx = \frac{x}{8}(2x^2-a^2)\sqrt{a^2-x^2} + \frac{a^4}{8}\sin^{-1}\left(\frac{x}{a}\right)$

5. $\int \frac{\sqrt{a^2-x^2}}{x}dx = \sqrt{a^2-x^2} - a\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right|$

6. $\int \frac{\sqrt{a^2-x^2}}{x^2}dx = -\frac{1}{x}\sqrt{a^2-x^2} - \sin^{-1}\left(\frac{x}{a}\right)$

7. $\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right)$

8. $\int \frac{x}{\sqrt{a^2-x^2}}dx = -\sqrt{a^2-x^2}$

9. $\int \frac{x^2}{\sqrt{a^2-x^2}}dx = -\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$

10. $\int \frac{1}{x\sqrt{a^2-x^2}}dx = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right|$

11. $\int \frac{1}{x^2\sqrt{a^2-x^2}}dx = -\frac{1}{a^2x}\sqrt{a^2-x^2}$

12. $\int (a^2-x^2)^{3/2}dx = -\frac{x}{8}(2x^2-5a^2)\sqrt{a^2-x^2} + \frac{3a^4}{8}\sin^{-1}\left(\frac{x}{a}\right)$

13. $\int \frac{1}{(a^2-x^2)^{3/2}}dx = \frac{x}{a^2\sqrt{a^2-x^2}}$

Integrals Involving $x^2 \pm a^2$ $\sqrt{x^2 \pm a^2}$

Note: $\ln\left|x+\sqrt{x^2+a^2}\right| = \sinh^{-1}\left(\frac{x}{a}\right)$,

$\ln\left|x+\sqrt{x^2-a^2}\right| = \cosh^{-1}\left(\frac{x}{a}\right)$, $\ln\left|\frac{a+\sqrt{x^2+a^2}}{x}\right| = \sinh^{-1}\left(\frac{a}{x}\right)$

$\int F(\sqrt{x^2+a^2})dx = a\int F(a\sec u)\sec^2 u du, x = a\tan u$

$\int F(\sqrt{x^2-a^2})dx = a\int F(a\tan u)\sec u\tan u du, x = a\sec u$

1. $\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

2. $\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|$

3. $\int \frac{x}{x^2+a^2}dx = \frac{1}{2}\ln|x^2+a^2|$

4. $\int \sqrt{x^2 \pm a^2}dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2}\ln\left|x+\sqrt{x^2 \pm a^2}\right|$

5. $\int x\sqrt{x^2 \pm a^2}dx = \frac{1}{3}(x^2 \pm a^2)^{3/2}$

6. $\int x^2\sqrt{x^2 \pm a^2}dx = \frac{x}{8}(2x^2 \pm a^2)\sqrt{x^2 \pm a^2} - \frac{a^4}{8}\ln\left|x+\sqrt{x^2 \pm a^2}\right|$

7. $\int \frac{\sqrt{x^2+a^2}}{x}dx = \sqrt{x^2+a^2} - a\sinh^{-1}\left(\frac{a}{x}\right)$

8. $\int \frac{\sqrt{x^2-a^2}}{x}dx = \sqrt{x^2-a^2} - a\sec^{-1}\left|\frac{x}{a}\right|$

9. $\int \frac{\sqrt{x^2 \pm a^2}}{x^2}dx = -\frac{\sqrt{x^2 \pm a^2}}{x} + \ln\left|x+\sqrt{x^2 \pm a^2}\right|$

10. $\int \frac{1}{\sqrt{x^2 \pm a^2}}dx = \ln\left|x+\sqrt{x^2 \pm a^2}\right|$

11. $\int \frac{1}{x^2\sqrt{x^2 \pm a^2}}dx = \mp\frac{\sqrt{x^2 \pm a^2}}{a^2x}$

12. $\int \frac{1}{x\sqrt{x^2+a^2}}dx = -\frac{1}{a}\sinh^{-1}\left(\frac{a}{x}\right)$

13. $\int \frac{1}{x\sqrt{x^2-a^2}}dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right|$

14. $\int \frac{x}{\sqrt{x^2 \pm a^2}}dx = \sqrt{x^2 \pm a^2}$

15. $\int \frac{x^2}{\sqrt{x^2 \pm a^2}}dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \mp \frac{a^2}{2}\ln\left|x+\sqrt{x^2 \pm a^2}\right|$

16. $\int (x^2 \pm a^2)^{3/2}dx = \frac{x}{8}(2x^2 \pm 5a^2)\sqrt{x^2 \pm a^2} + \frac{3a^4}{8}\ln\left|x+\sqrt{x^2 \pm a^2}\right|$

17. $\int \frac{1}{(x^2 \pm a^2)^{3/2}}dx = \pm\frac{x}{a^2\sqrt{x^2 \pm a^2}}$

Integrals Involving $ax^2 + bx + c$

1. $\int \frac{1}{ax^2+c}dx = \frac{1}{\sqrt{ac}}\tan^{-1}\left(x\sqrt{\frac{a}{c}}\right)$

2a. $\int \frac{1}{ax^2+bx+c}dx = \frac{2}{\sqrt{4ac-b^2}}\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)$ **for** $4ac - b^2 > 0$

2b. $\int \frac{1}{ax^2+bx+c}dx = \frac{1}{\sqrt{b^2-4ac}}\ln\left|\frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right|$ **for** $b^2 - 4ac > 0$

2c. $\int \frac{1}{ax^2+bx+c}dx = -\frac{2}{2ax+b}$ $\quad b^2 - 4ac = 0$

Integrals Involving $a^2 \pm b^2x^2$

1. $\int \frac{1}{a^2+b^2x^2}dx = \frac{1}{ab}\tan^{-1}\frac{bx}{a} \quad (a>0, b>0)$

2. $\int \frac{1}{a^2-b^2x^2}dx = \frac{1}{ab}\tanh^{-1}\frac{bx}{a} = \frac{1}{2ab}\ln\left(\frac{a+bx}{a-bx}\right)$ **for** $(a>0, b>0)$

3. $\int \sqrt{a^2+b^2x^2}dx = \frac{x}{2}\sqrt{a^2+b^2x^2} + \frac{a^2}{2b}\ln\left(\frac{bx}{a} + \sqrt{1+\frac{b^2x^2}{a^2}}\right)$

4. $\int \frac{1}{\sqrt{a^2+b^2x^2}}dx = \frac{1}{b}\ln\left(\frac{bx}{a} + \sqrt{1+\frac{b^2x^2}{a^2}}\right)$

5. $\int \sqrt{a^2-b^2x^2}dx = \frac{x}{2}\sqrt{a^2-b^2x^2} + \frac{a^2}{2b}\sin^{-1}\frac{bx}{a}$

6. $\int \frac{1}{\sqrt{a^2-b^2x^2}}dx = \frac{1}{b}\sin^{-1}\frac{bx}{a}$

Integrals Involving $\sqrt{2ax - x^2}$

1. $\int \sqrt{2ax-x^2}dx = \frac{x-a}{2}\sqrt{2ax-x^2} + \frac{a^2}{2}\cos^{-1}\left(1-\frac{x}{a}\right)$

2. $\int x\sqrt{2ax-x^2}dx = \frac{2x^2-ax-3a^2}{6}\sqrt{2ax-x^2} + \frac{a^3}{2}\cos^{-1}\left(1-\frac{x}{a}\right)$

3. $\int \frac{\sqrt{2ax-x^2}}{x}dx = \sqrt{2ax-x^2} + a\cos^{-1}\left(1-\frac{x}{a}\right)$

4. $\int \frac{\sqrt{2ax-x^2}}{x^2}dx = -\frac{2\sqrt{2ax-x^2}}{x} - \cos^{-1}\left(1-\frac{x}{a}\right)$

5. $\int \frac{1}{\sqrt{2ax-x^2}}dx = \cos^{-1}\left(1-\frac{x}{a}\right)$

6. $\int \frac{x}{\sqrt{2ax-x^2}}dx = -\sqrt{2ax-x^2} + a\cos^{-1}\left(1-\frac{x}{a}\right)$

7. $\int \frac{x^2}{\sqrt{2ax-x^2}}dx = -\frac{x+3a}{2}\sqrt{2ax-x^2} + \frac{3a^2}{2}\cos^{-1}\left(1-\frac{x}{a}\right)$

8. $\int \frac{1}{x\sqrt{2ax-x^2}}dx = -\frac{\sqrt{2ax-x^2}}{ax}$

9. $\int \frac{1}{(2ax-x^2)^{3/2}}dx = \frac{x-a}{a^2\sqrt{2ax-x^2}}$

Miscellaneous Integrals

1. $\int x\sqrt{\frac{a+x}{a-x}}dx = -\frac{2a+x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}$

2. $\int \sqrt{\frac{x}{x^3-a}}dx = \frac{2}{3}\ln(x^{3/2} + \sqrt{x^3-a})$

3. $\int \sqrt{\frac{x}{x+a}}dx = \sqrt{x^2+ax} - a\ln(\sqrt{x+a} + \sqrt{x})$

4. $\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}dx = \frac{1}{\sqrt{2}}\ln\frac{\sqrt{2}x + \sqrt{1+x^4}}{1-x^2}$

DERIVATIVES

$\frac{dw}{dx} = \frac{dw}{du}\frac{du}{dx}$ **: Chain Rule** $\quad \frac{du}{dx} = \left(\frac{dx}{du}\right)^{-1} = \frac{1}{dx/du}$

$\frac{d}{dx}f(u) = \frac{du}{dx}\frac{d}{du}f(u)$ $\quad \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$ $\quad \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{1}{v^2}\left(v\frac{du}{dx} - u\frac{dv}{dx}\right)$

$\frac{d}{dx}(u^n) = vu^{n-1}\frac{du}{dx} + u^n\ln u\frac{dv}{dx}$

$\frac{d}{dx}\ln x = \frac{1}{x}$ $\quad \frac{d}{dx}e^x = e^x$

$\frac{d}{dx}a^x = a^x\ln a$ $\quad \frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$

$\frac{d(\log_a u)}{dx} = (\log_a e)\frac{1}{u}\frac{du}{dx}$

$d(\sin x)/dx = \cos x$ $\quad d(\cos x)/dx = -\sin x$

$d(\tan x)/dx = \sec^2 x$ $\quad d(\cot x)/dx = -\csc^2 x$

$d(\sec x)/dx = \sec x\tan x$ $\quad d(\csc x)/dx = -\csc x\cot x$

$\frac{d(\sin^{-1}u)}{dx} = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$ **for** $\left(-\frac{\pi}{2} \le \sin^{-1}u \le \frac{\pi}{2}\right)$

$\frac{d(\cos^{-1}u)}{dx} = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$ **for** $(0 \le \cos^{-1}u \le \pi)$

$\frac{d(\tan^{-1}u)}{dx} = \frac{1}{1+u^2}\frac{du}{dx}$ **for** $\left(-\frac{\pi}{2} < \tan^{-1}u < \frac{\pi}{2}\right)$

$\frac{d(\cot^{-1}u)}{dx} = \frac{-1}{1+u^2}\frac{du}{dx}$ **for** $(0 < \cot^{-1}u < \pi)$

$\frac{d(\sec^{-1}u)}{dx} = \frac{1}{u\sqrt{u^2-1}}\frac{du}{dx}$ $\left(0 \le \sec^{-1}u < \frac{\pi}{2}\right)\left(-\pi \le \sec^{-1}u < -\frac{\pi}{2}\right)$

$\frac{d(\csc^{-1}u)}{dx} = \frac{-1}{u\sqrt{u^2-1}}\frac{du}{dx}$ $\left(0 < \csc^{-1}u \le \frac{\pi}{2}\right)\left(-\pi < \csc^{-1}u \le -\frac{\pi}{2}\right)$

$d(\sinh x)/dx = \cosh x$ $\quad d(\cosh x)/dx = \sinh x$

$d(\tanh x)/dx = \operatorname{sech}^2 x$ $\quad d(\coth x)/dx = -\operatorname{csch}^2 x$

$d(\operatorname{sech} x)/dx = -\operatorname{sech} x\tanh x$

$d(\operatorname{csch} x)/dx = -\operatorname{csch} x\coth x$

$\frac{d}{dx}\sinh^{-1}u = \frac{1}{\sqrt{u^2+1}}\frac{du}{dx}$

$\frac{d}{dx}\cosh^{-1}u = \frac{1}{\sqrt{u^2-1}}\frac{du}{dx}$ **for** $u > 1$ **and** $\cosh^{-1}u \ge 0$

$\frac{d}{dx}\tanh^{-1}u = \frac{1}{1-u^2}\frac{du}{dx}$ **for** $-1 < u < 1$