TRIGONOMETRY REVIEW SHEET ©2005 Jon Wittwer, www.vertex42.com

QUADRANTS & SIGNS OF FUNCTIONS

sin & csc pos. others neg. **II**	**I** All positive
III tan & cot pos. others neg.	**IV** cos & sec pos. others neg.

RIGHT-ANGLE TRIANGLE RELATIONSHIPS
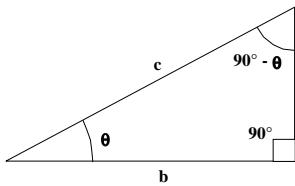
a = opposite side
b = adjacent side
c = hypotenuse

$$\sin\theta = \frac{\text{side opposite }\theta}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos\theta = \frac{\text{side adjacent to }\theta}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan\theta = \frac{\text{side opposite }\theta}{\text{side adjacent to }\theta} = \frac{a}{b}$$

Pythagorean theorem: $c^2 = a^2 + b^2$

FUNDAMENTAL IDENTITIES

1. $\tan x = \dfrac{\sin x}{\cos x}$ 2. $\sec x = \dfrac{1}{\cos x}$ 3. $\csc x = \dfrac{1}{\sin x}$ 4. $\cot x = \dfrac{1}{\tan x} = \dfrac{\cos x}{\sin x}$

5. $\cos^2 x + \sin^2 x = 1$ 6. $1 + \tan^2 x = \sec^2 x$ 7. $\cot^2 x + 1 = \csc^2 x$

8a. $\cos(\pi/2 - x) = \sin x$ 8b. $\cos(\pi/2 + x) = -\sin x$

9a. $\sin(\pi/2 - x) = \cos x$ 9b. $\sin(\pi/2 + x) = \cos x$

10. $\tan(\pi/2 - x) = \cot x$

OPPOSITE-ANGLE IDENTITIES

1. $\cos(-x) = \cos x$ 2. $\sin(-x) = -\sin x$ 3. $\tan(-x) = -\tan x$

4. $\sec(-x) = \sec x$ 5. $\csc(-x) = -\csc x$ 6. $\cot(-x) = -\cot x$

ADDITION LAWS

1. $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$ 2. $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$

3. $\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

DOUBLE-ANGLE IDENTITIES

1a. $\cos 2x = \cos^2 x - \sin^2 x$ 1b. $\cos 2x = 2\cos^2 x - 1$ 1c. $\cos 2x = 1 - 2\sin^2 x$

1d. $\sin^2 x = \dfrac{1}{2}(1 - \cos 2x)$ 1e. $\cos^2 x = \dfrac{1}{2}(1 + \cos 2x)$

2. $\sin 2x = 2\sin x \cos x$ 3. $\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

HALF-ANGLE IDENTITIES

1. $\cos\dfrac{x}{2} = \pm\sqrt{\dfrac{1 + \cos x}{2}}$ $\begin{cases}+ \text{ if x/2 is in quadrant I or IV}\\ - \text{ if x/2 is in quadrant II or III}\end{cases}$

2. $\sin\dfrac{x}{2} = \pm\sqrt{\dfrac{1 - \cos x}{2}}$ $\begin{cases}+ \text{ if x/2 is in quadrant I or II}\\ - \text{ if x/2 is in quadrant III or IV}\end{cases}$

3a. $\tan\dfrac{x}{2} = \pm\sqrt{\dfrac{1 - \cos x}{1 + \cos x}}$ $\begin{cases}+ \text{ if x/2 is in quadrant I or III}\\ - \text{ if x/2 is in quadrant II or IV}\end{cases}$

3b. $\tan\dfrac{x}{2} = \dfrac{1 - \cos x}{\sin x} = \dfrac{\sin x}{1 + \cos x} = \csc x - \cot x$

PRODUCT IDENTITIES

1. $2\cos x \cos y = \cos(x - y) + \cos(x + y)$ 2. $2\sin x \sin y = \cos(x - y) - \cos(x + y)$

3. $2\sin x \cos y = \sin(x + y) + \sin(x - y)$ 4. $2\cos x \sin y = \sin(x + y) - \sin(x - y)$

5. $\cos mx \cos nx = \cos(m + n)x + \cos(m - n)x$

SUM IDENTITIES

1. $\cos x + \cos y = 2\cos\left(\dfrac{x + y}{2}\right)\cos\left(\dfrac{x - y}{2}\right)$ 2. $\cos x - \cos y = -2\sin\left(\dfrac{x + y}{2}\right)\sin\left(\dfrac{x - y}{2}\right)$

3. $\sin x + \sin y = 2\sin\left(\dfrac{x + y}{2}\right)\cos\left(\dfrac{x - y}{2}\right)$ 4. $\sin x - \sin y = 2\sin\left(\dfrac{x - y}{2}\right)\cos\left(\dfrac{x + y}{2}\right)$

REDUCTION IDENTITY

1. $a\sin x + b\cos x = \sqrt{a^2 + b^2}\sin(x + y)$, where y is chosen so that

$\cos y = \dfrac{a}{\sqrt{a^2 + b^2}}$ and $\sin y = \dfrac{b}{\sqrt{a^2 + b^2}}$

PLANE TRIANGLE RELATIONSHIPS
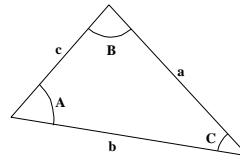
Law of sines: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Law of cosines: $c^2 = a^2 + b^2 - 2ab\cos C$,

$C = \mathrm{acos}\left(\dfrac{a^2 + b^2 - c^2}{2ab}\right)$

Law of tangents: $\dfrac{a + b}{a - b} = \dfrac{\tan\frac{1}{2}(A + B)}{\tan\frac{1}{2}(A - B)}$

INVERSE TRIGONOMETRIC FUNCTIONS

Function	Domain	Range	Quadrants
$y = \cos^{-1}x$	$-1 \le x \le 1$	$0 \le y \le \pi$	I and II
$y = \sin^{-1}x$	$-1 \le x \le 1$	$-\pi/2 \le y \le \pi/2$	I and IV
$y = \tan^{-1}x$	all reals	$-\pi/2 < y < \pi/2$	I and IV
$y = \sec^{-1}x$	$x \ge 1$ or $x \le -1$	$0 \le y \le \pi$, $y \ne \pi/2$	I and II
$y = \csc^{-1}x$	$x \ge 1$ or $x \le -1$	$-\pi/2 \le y \le \pi/2$, $y \ne 0$	I and IV
$y = \cot^{-1}x$	all reals	$0 < y < \pi$	I and II

PRINCIPAL VALUES FOR INVERSE TRIGONOMETRIC FUNCTIONS

Principal values for $x \ge 0$	Principal values for $x < 0$
$0 \le \sin^{-1}x \le \pi/2$	$-\pi/2 \le \sin^{-1}x < 0$
$0 \le \cos^{-1}x \le \pi/2$	$\pi/2 < \cos^{-1}x \le \pi$
$0 \le \tan^{-1}x < \pi/2$	$-\pi/2 < \tan^{-1}x < 0$
$0 < \cot^{-1}x \le \pi/2$	$\pi/2 < \cot^{-1}x < \pi$
$0 \le \sec^{-1}x < \pi/2$	$\pi/2 < \sec^{-1}x \le \pi$
$0 < \csc^{-1}x \le \pi/2$	$-\pi/2 \le \csc^{-1}x < 0$

INVERSE IDENTITIES (ASSUMING PRINCIPAL VALUES ARE USED)

1. $\sin^{-1}x + \cos^{-1}x = \pi/2$ 7. $\sin^{-1}(-x) = -\sin^{-1}x$

2. $\tan^{-1}x + \cot^{-1}x = \pi/2$ 8. $\cos^{-1}(-x) = \pi - \cos^{-1}x$

3. $\sec^{-1}x + \csc^{-1}x = \pi/2$ 9. $\tan^{-1}(-x) = -\tan^{-1}x$

4. $\csc^{-1}x = \sin^{-1}(1/x)$ 10. $\cot^{-1}(-x) = \pi - \cot^{-1}x$

5. $\sec^{-1}x = \cos^{-1}(1/x)$ 11. $\sec^{-1}(-x) = \pi - \sec^{-1}x$

6. $\cot^{-1}x = \tan^{-1}(1/x)$ 12. $\csc^{-1}(-x) = -\csc^{-1}x$

COMPLEX IDENTITIES

$\cos w = \dfrac{e^{iw} + e^{-iw}}{2}$ $i\sin w = \dfrac{e^{iw} - e^{-iw}}{2}$ Euler Identity: $e^{iw} = \cos w + i\sin w$

QUADRATIC FORMULA

Solution to $ax^2 + bx + c = 0$: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

EXACT VALUES

Angle (deg)	Angle (rad)	cos(x)	sin(x)	tan(x)
0°	0	1	0	0
30°	$\pi/6$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}/3$
45°	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60°	$\pi/3$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}$
90°	$\pi/2$	0	1	$\pm\infty$
180°	π	-1	0	0
270°	$3\pi/2$	0	-1	$\pm\infty$