

Note: u, v, and w are functions of x, a, c, n, are constants. All trigonometric functions use radians. A constant is still added to the result of every integration.

General and Basic Integrals

- 1. ∫ f(x) dx = a f(x) dx
2. ∫ (u v) dx = ∫ u dx ∫ v dx
3. ∫ u dv = uv - ∫ v du
5. ∫ g'(x) dx = ln|g(x)|
6. ∫ [f(x)g'(x)] dx = { [(f(x))^(r+1)] / (r+1) - r ≠ 1
ln|g(x)| r = 1
7. ∫ x^n dx = x^(n+1) / (n+1) (n ≠ -1)
8. ∫ dx/x = ∫ x^-1 dx = ln|x| (x ≠ 0)
9. ∫ dx/x^n = ∫ x^-n dx = x^(1-n) / (1-n) (n ≠ 1)

SIN

- 1. ∫ sin x dx = -cos x
2. ∫ sin ax dx = -1/a cos ax
3. ∫ sin^2 x dx = 1/2 x - 1/4 sin 2x
4. ∫ sin^2 ax dx = 1/2 x - 1/4 sin 2ax
5. ∫ sin^3 x dx = -1/3 sin^2 x cos x - 1/3 sin x cos^2 x + 1/3 x
6. ∫ sin^n x dx = -1/n sin^(n-1) x cos x + (n-1)/n ∫ sin^(n-2) x dx
7. ∫ x sin x dx = sin x - x cos x
8. ∫ x sin ax dx = sin ax/x - x cos ax/a
9. ∫ x^2 sin x dx = -x^2 cos x + 2x sin x + 2 cos x
10. ∫ x^n sin x dx = -x^n cos x + n ∫ x^(n-1) cos x dx
11. ∫ sin^-1 x dx = x sin^-1 x + √(1-x^2)
12. ∫ sin^-1 ax dx = x sin^-1 ax + 1/a √(1-a^2 x^2)

COS

- 1. ∫ cos x dx = sin x
2. ∫ cos ax dx = 1/a sin ax
3. ∫ cos^2 x dx = 1/2 x + 1/4 sin 2x
4. ∫ cos^2 ax dx = 1/2 x + 1/4 sin 2ax
5. ∫ cos^3 x dx = 1/4 cos^3 x sin x + 3/8 cos x sin x + 3/8 x
6. ∫ cos^n x dx = 1/n cos^(n-1) x sin x + (n-1)/n ∫ cos^(n-2) x dx
7. ∫ x cos x dx = cos x + x sin x
8. ∫ x cos ax dx = cos ax/x + x sin ax/a
9. ∫ x^2 cos x dx = x^2 sin x + 2x cos x - 2 sin x
10. ∫ x^n cos x dx = x^n sin x - n ∫ x^(n-1) sin x dx
11. ∫ cos^-1 x dx = x cos^-1 x - √(1-x^2)
12. ∫ cos^-1 ax dx = x cos^-1 ax - 1/a √(1-a^2 x^2)

TAN

- 1. ∫ tan x dx = -ln|cos x|
2. ∫ tan ax dx = -1/a ln|cos ax|
3. ∫ tan^2 x dx = tan x - x
4. ∫ tan^2 ax dx = x - 1/a tan ax
5. ∫ tan^n x dx = 1/(n-1) tan^(n-1) x - ∫ tan^(n-2) x dx
6. ∫ tan^-1 x dx = x tan^-1 x - 1/2 ln|x^2+1|
7. ∫ 1/(1+tan x) dx = 1/2 [x + 1/a ln|cos ax + sin ax|]

COT

- 1. ∫ cot x dx = ln|sin x|
2. ∫ cot ax dx = 1/a ln|sin ax|
3. ∫ cot^2 x dx = -cot x - x
4. ∫ cot^2 ax dx = -x - 1/a cot ax
5. ∫ cot^n x dx = -1/(n-1) cot^(n-1) x - ∫ cot^(n-2) x dx
6. ∫ cot^-1 x dx = x cot^-1 x + 1/2 ln|x^2+1|
7. ∫ 1/(1+cot x) dx = 1/2 [x + 1/a ln|cos ax + sin ax|]

SEC

- 1. ∫ sec x dx = ln|sec x + tan x|
2. ∫ sec ax dx = 1/a ln|sec ax + tan ax|
3. ∫ sec^2 x dx = tan x
4. ∫ sec^3 x dx = 1/2 sec x tan x + 1/2 ln|sec x + tan x|
5. ∫ sec^n x dx = 1/(n-1) sec^(n-2) x tan x + (n-2)/(n-1) ∫ sec^(n-2) x dx
6. ∫ sec^-1 x dx = x sec^-1 x - ln|x + √(x^2-1)|

CSC

- 1. ∫ csc x dx = -ln|csc x + cot x|
2. ∫ csc ax dx = -1/a ln|csc ax + cot ax|
3. ∫ csc^2 x dx = -cot x

- 4. ∫ csc^n x dx = -1/(n-1) csc^(n-2) x cot x + (n-2)/(n-1) ∫ csc^(n-2) x dx
5. ∫ csc^-1 x dx = x csc^-1 x + ln|x + √(x^2-1)|

Combined Trig Functions

- 1. ∫ sin x cos x dx = (sin^2 x)/2
2. ∫ sin ax cos bx dx = -cos(a-b)x / (2(a-b)) - cos(a+b)x / (2(a+b))
3. ∫ sec x tan x dx = sec x
4. ∫ csc x cot x dx = -csc x
5a. ∫ sin^m x cos^n x dx = sin^(m-1) x cos^(n-1) x + (n-1)/m+n ∫ sin^m x cos^(n-2) x dx
∫ sin^m x cos^n x dx = -sin^(m-1) x cos^(n+1) x / (m+n) + (m-1)/m+n ∫ sin^(m-2) x cos^n x dx
6. ∫ 1/(a sin x + b cos x) dx = 1/(c√(a^2+b^2)) ln|tan(1/2)(x + tan^-1(b/a))|

Integrals Involving e^x

- ∫ F(e^x) dx = 1/∫ F(u) du, u = e^x
1. ∫ e^x dx = e^x
2. ∫ e^ax dx = 1/a e^ax
3. ∫ x e^x dx = x e^x - e^x
4. ∫ x^n e^x dx = e^x (x^n - n x^(n-1) + n(n-1) x^(n-2) - ... + (-1)^(n-1) x + (-1)^n)
5. ∫ x^n e^ax dx = x^n e^ax / a - n/a ∫ x^(n-1) e^ax dx
6. ∫ dx/(a+be^x) = x/a - 1/ac ln|a+be^x|
7. ∫ (a+e^x)/b dx = ax/b + e^x/b ln|b+e^x|
8. ∫ (a+be^x+c e^2x)/d dx = ax/d + ce^x/d - a-bd+cd^2/d ln|d+e^x|
9. ∫ x/(e^x-1) dx = -1/e^x + 1
10. ∫ √(1+e^x) dx = 2/3 √(1+e^x) + 1/3 ln|1+√(1+e^x)|
11. ∫ 1/√(e^x+a^2) dx = 2/√(a^2+1) ln|√(e^x+a^2+1) + √(e^x+a^2)|

Integrals Involving a^x

- 1. ∫ a^x dx = a^x / ln a
2. ∫ x a^x dx = x a^x / ln a - a^x / (ln a)^2
3. ∫ x^n a^x dx = x^n a^x / ln a - n/a ∫ x^(n-1) a^x dx
4. ∫ x^n a^x dx = x^n a^x / ln a - n/a ∫ x^(n-1) a^x dx

Integrals Involving ln(x)

- ∫ F(ln x) dx = ∫ F(u) e^u du, u = ln x
1. ∫ ln x dx = -x + x ln x
2. ∫ ln ax dx = -x + x ln ax
3. ∫ (ln x)^2 dx = x(ln x)^2 - 2x ln x + 2x
4. ∫ 1/x dx = ln|x|
5. ∫ 1/x^n dx = ln|ln|x||
6. ∫ x^n ln x dx = 1/(n+1) x^(n+1) ln x - 1/(n+1)^2 x^(n+1)
7. ∫ e^ax sin bx dx = e^ax/(a^2+b^2) (a sin bx - b cos bx)
8. ∫ e^ax cos bx dx = e^ax/(a^2+b^2) (a cos bx + b sin bx)

Hyperbolic Functions

- Note: sinh x = (e^x - e^-x)/2, cosh x = (e^x + e^-x)/2, tanh x = sinh x / cosh x
1. ∫ sinh x dx = cosh x
2. ∫ sinh^2 x dx = 1/4 sinh 2x - 1/4 x
3. ∫ cosh x dx = sinh x
4. ∫ cosh^2 x dx = 1/4 sinh 2x + 1/4 x
5. ∫ tanh x dx = ln|cosh x|
6. ∫ coth x dx = ln|sinh x|
7. ∫ sech x dx = tan^-1(sinh x)
8. ∫ sech^2 x dx = tanh x
9. ∫ sech x tanh x dx = -sech x
10. ∫ csch x coth x dx = -csch x
11. ∫ csch x dx = ln|tanh(1/2)x|
12. ∫ e^ax sinh bx dx = e^ax/(a^2-b^2) (a sinh bx - b cosh bx)
13. ∫ e^ax cosh bx dx = e^ax/(a^2-b^2) (a cosh bx + b sinh bx)

Integrals Involving a+bx

- ∫ F(a+bx) dx = 1/b ∫ F(u) du, u = a+bx
1. ∫ 1/(a+bx) dx = 1/b ln|a+bx|
2. ∫ 1/(a+bx)^2 dx = -1/(a+bx)
3. ∫ 1/(a+bx)^n dx = -1/((n-1)b(a+bx)^(n-1)) (n ≠ 1)
4. ∫ x/(a+bx) dx = 1/b [x + (a-b)/b ln|a+bx|]
5. ∫ x/(a+bx)^2 dx = 1/b^2 [x + (a-b)/b ln|a+bx|]
6. ∫ x/(a+bx)^3 dx = -1/(2b^2(a+bx)) + 1/b ln|a+bx|
7. ∫ x^2/(a+bx) dx = 1/b^2 [1/2 (a+bx)^2 - 2a(a+bx) + a^2 ln|a+bx|]
8. ∫ x^2/(a+bx)^2 dx = 1/b [x + (a-b)/b ln|a+bx|]
9. ∫ (a+bx)^n dx = (a+bx)^(n+1) / (b(n+1)) (n ≠ -1)
10. ∫ 1/(x(a+bx)) dx = 1/a ln|a/x| + 1/b ln|a+bx|
11. ∫ 1/(x^2(a+bx)) dx = -1/a x - b/a ln|a+bx|
12. ∫ 1/(x(a+bx)^2) dx = 1/a^2 ln|a/x| + 1/a ln|a+bx|

Integrals Involving √(a+bx)

- ∫ F(√(a+bx)) dx = 2/√b ∫ F(u) du, u = √(a+bx)
∫ F(√(a+bx)) dx = 2/√b ∫ F(u) du, u = √(a+bx)
∫ F(√(a+bx)) dx = 2/√b ∫ F(u) du, u = √(a+bx)

- 1. ∫ √(a+bx) dx = 2/(3b) (a+bx)^(3/2)
2. ∫ x√(a+bx) dx = 2/(15b^2) (3bx-2a)(a+bx)^(3/2)
3. ∫ x^n √(a+bx) dx = 2x^n (a+bx)^(3/2) / (b(2n+3)) - 2an/(b(2n+3)) x^(n-1) √(a+bx)
4. ∫ x/√(a+bx) dx = 2/(3b^2) (bx-2a)√(a+bx)
5. ∫ x^n/√(a+bx) dx = 2x^n √(a+bx) / (b(2n+1)) - 2an/(b(2n+1)) ∫ x^(n-1)/√(a+bx) dx
6. ∫ 1/√(a+bx) dx = { 1/√a ln|√(a+bx) + √a| for (a > 0)
2/√(-a) tan^-1(√(a+bx)/√(-a)) for (a < 0)
7. ∫ 1/(x√(a+bx)) dx = -1/(a(n-1)x^(n-1)) - b(2n-3)/(2a(n-1)) ∫ 1/(x√(a+bx)) dx
8. ∫ √(a+bx)/x dx = 2√(a+bx) + a ∫ 1/√(a+bx) dx
9. ∫ √(a+bx)/x^n dx = (a+bx)^(3/2) / (a(n-1)x^(n-1)) - b(2n-5)/(2a(n-1)) ∫ √(a+bx)/x^n dx
10. ∫ 1/(√(a+bx)√(c+dx)) dx = 2/√ac tanh^-1(c/(ax+b)) / √(a(cx+d))

Integrals Involving a^2-x^2

- ∫ F(√(a^2-x^2)) dx = a ∫ F(a cos u) cos u du, x = a sin u
1. ∫ 1/(a^2-x^2) dx = 1/a ln|(x+a)/(x-a)|
2. ∫ √(a^2-x^2) dx = x/2 √(a^2-x^2) + a^2/2 sin^-1(x/a)
3. ∫ x√(a^2-x^2) dx = -1/3 (a^2-x^2)^(3/2)
4. ∫ x^2√(a^2-x^2) dx = x/8 (2x^2-a^2)√(a^2-x^2) + a^4/8 sin^-1(x/a)
5. ∫ √(a^2-x^2)/x dx = √(a^2-x^2) - a ln|a+√(a^2-x^2)|
6. ∫ √(a^2-x^2)/x^2 dx = -1/x √(a^2-x^2) - sin^-1(x/a)
7. ∫ 1/(x√(a^2-x^2)) dx = sin^-1(x/a)
8. ∫ x/√(a^2-x^2) dx = -√(a^2-x^2)
9. ∫ √(a^2-x^2)/x^2 dx = x/2 √(a^2-x^2) + a^2/2 sin^-1(x/a)
10. ∫ 1/(x√(a^2-x^2)) dx = -1/a ln|(a+√(a^2-x^2))/x|
11. ∫ 1/(x^2√(a^2-x^2)) dx = 1/a^2 √(a^2-x^2)
12. ∫ (a^2-x^2)^(3/2) dx = -x/8 (2x^2-5a^2)√(a^2-x^2) + 3a^4/8 sin^-1(x/a)
13. ∫ 1/(a^2-x^2)^(3/2) dx = x/(a^2√(a^2-x^2))

Integrals Involving x^2±a^2

- Note: ln|x+√(x^2+a^2)| = sinh^-1(x/a)
ln|x+√(x^2-a^2)| = cosh^-1(x/a)
∫ F(√(x^2+a^2)) dx = a ∫ F(a sec u) sec^3 u du, x = a tan u
∫ F(√(x^2-a^2)) dx = a ∫ F(a tan u) sec u tan u du, x = a sec u
1. ∫ 1/(x^2+a^2) dx = 1/a tan^-1(x/a)
2. ∫ 1/(x^2-a^2) dx = 1/a ln|(x-a)/(x+a)|
3. ∫ x/(x^2+a^2) dx = 1/2 ln|x^2+a^2|
4. ∫ √(x^2+a^2) dx = x/2 √(x^2+a^2) + a^2/2 ln|x+√(x^2+a^2)|
5. ∫ x√(x^2+a^2) dx = 1/3 (x^2+a^2)^(3/2)
6. ∫ x^2√(x^2+a^2) dx = x/8 (2x^2+a^2)√(x^2+a^2) - a^4/8 ln|x+√(x^2+a^2)|
7. ∫ √(x^2+a^2)/x dx = √(x^2+a^2) - a sinh^-1(x/a)
8. ∫ √(x^2-a^2)/x dx = √(x^2-a^2) - a sec^-1|x/a|
9. ∫ √(x^2+a^2)/x^2 dx = -√(x^2+a^2)/x + ln|x+√(x^2+a^2)|
10. ∫ 1/√(x^2+a^2) dx = ln|x+√(x^2+a^2)|
11. ∫ 1/(x^2√(x^2+a^2)) dx = -√(x^2+a^2)/a^2 x
12. ∫ 1/(x√(x^2+a^2)) dx = 1/a sec^-1|x/a|
13. ∫ x/√(x^2+a^2) dx = √(x^2+a^2)
14. ∫ x/√(x^2-a^2) dx = √(x^2-a^2)
15. ∫ x^2/√(x^2+a^2) dx = x/2 √(x^2+a^2) + a^2/2 ln|x+√(x^2+a^2)|
16. ∫ (x^2+a^2)^(3/2) dx = x/8 (2x^2+5a^2)√(x^2+a^2) + 3a^4/8 ln|x+√(x^2+a^2)|
17. ∫ 1/(x^2±a^2)^(3/2) dx = ±x/(a^2√(x^2±a^2))

Integrals Involving ax^2+bx+c

- 1. ∫ 1/(ax^2+c) dx = 1/√ac tan^-1(x/√(c/a))
2a. ∫ 1/(ax^2+bx+c) dx = 2/√(4ac-b^2) tan^-1((2ax+b)/√(4ac-b^2)) for 4ac-b^2 > 0
2b. ∫ 1/(ax^2+bx+c) dx = 1/√(b^2-4ac) ln|(2ax+b-√(b^2-4ac))/(2ax+b+√(b^2-4ac))| for b^2-4ac > 0

- 2c. ∫ 1/(ax^2+bx+c) dx = 2/(2ax+b) - b^2-4ac = 0

Integrals Involving a^2±b^2x^2

- 1. ∫ 1/(a^2+b^2x^2) dx = 1/a tan^-1(bx/a) (a > 0, b > 0)
2. ∫ 1/(a^2-b^2x^2) dx = 1/a tan^-1(bx/a) = 1/(2ab) ln|(a+bx)/(a-bx)| for (a > 0, b > 0)
3. ∫ √(a^2+b^2x^2) dx = x/2 √(a^2+b^2x^2) + a^2/2b ln|(bx/a + √(1+b^2x^2/a^2))|
4. ∫ 1/√(a^2+b^2x^2) dx = 1/b ln|(bx/a + √(1+b^2x^2/a^2))|
5. ∫ √(a^2-b^2x^2) dx = x/2 √(a^2-b^2x^2) + a^2/2b sin^-1(bx/a)
6. ∫ 1/√(a^2-b^2x^2) dx = 1/b sin^-1(bx/a)

Integrals Involving √(2ax-x^2)

- 1. ∫ √(2ax-x^2) dx = x/2 √(2ax-x^2) + a^2/2 cos^-1(1-x/a)
2. ∫ x√(2ax-x^2) dx = 2x^2-ax-3a^2/6 √(2ax-x^2) + a^3/2 cos^-1(1-x/a)
3. ∫ √(2ax-x^2)/x dx = √(2ax-x^2) + a cos^-1(1-x/a)
4. ∫ √(2ax-x^2)/x^2 dx = -2√(2ax-x^2)/x - cos^-1(1-x/a)
5. ∫ 1/√(2ax-x^2) dx = cos^-1(1-x/a)
6. ∫ x/√(2ax-x^2) dx = -√(2ax-x^2) + a cos^-1(1-x/a)
7. ∫ x^2/√(2ax-x^2) dx = -x+3a/2 √(2ax-x^2) + 3a^2/2 cos^-1(1-x/a)
8. ∫ 1/x√(2ax-x^2) dx = -√(2ax-x^2)/ax
9. ∫ 1/(2ax-x^2)^(3/2) dx = x-a/√(2ax-x^2)

Miscellaneous Integrals

- 1. ∫ x/(a-x) dx = -2a+x/2 √(2ax-x^2) + a^2 sin^-1(x/a)
2. ∫ √(x-a) dx = 2/3 ln|x^3+√(x^3-a)|
3. ∫ √(x+a) dx = √(x+a) - a ln|√(x+a)+√x|
4. ∫ 1/(1-x^2)√(1+x^2) dx = 1/√2 ln|√(2x+√(1+x^2))/(1-x^2)|

DERIVATIVES

- dw/dx = dw/du \* du/dx : Chain Rule
du/dx = (du/dt) \* dt/dx
d/dx(f(u)) = df/du \* du/dx
d/dx(uv) = u dv/dx + v du/dx
d/dx(u/v) = (v du/dx - u dv/dx)/v^2
d/dx(u^a) = a u^(a-1) du/dx
d/dx ln x = 1/x
d/dx a^x = a^x ln a
d/dx a^u = a^u ln a du/dx
d(log\_e u)/dx = (log\_e e) du/dx
d(sin x)/dx = cos x
d(tan x)/dx = sec^2 x
d(sec x)/dx = sec x tan x
d(sin^-1 u)/dx = 1/√(1-u^2) du/dx for (-π/2 ≤ sin^-1 u ≤ π/2)
d(cos^-1 u)/dx = -1/√(1-u^2) du/dx for (0 ≤ cos^-1 u ≤ π)
d(tan^-1 u)/dx = 1/(1+u^2) du/dx for (-π/2 < tan^-1 u < π/2)
d(cot^-1 u)/dx = -1/(1+u^2) du/dx for (0 < cot^-1 u < π)
d(sec^-1 u)/dx = 1/(u√(u^2-1)) du/dx for (0 ≤ sec^-1 u < π/2) (-π ≤ sec^-1 u < π/2)
d(csc^-1 u)/dx = -1/(u√(u^2-1)) du/dx for (0 < csc^-1 u ≤ π/2) (-π < csc^-1 u ≤ -π/2)
d(sinh x)/dx = cosh x
d(tanh x)/dx = sech^2 x
d(sech x)/dx = -sech x tanh x
d(csch x)/dx = -csch x coth x
d(sinh^-1 u)/dx = 1/√(u^2+1) du/dx
d(cosh^-1 u)/dx = 1/√(u^2-1) du/dx for u > 1 and cosh^-1 u ≥ 0
d(tanh^-1 u)/dx = 1/(1-u^2) du/dx for -1 < u < 1